

On the Gravitational Field of a Point-Mass, According to Einstein's Theory

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Abstract: This is a translation of the paper *Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie* by Karl Schwarzschild, where he obtained the metric of a space due to the gravitational field of a point-mass. The paper was originally published in 1916, in *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften*, S. 189–196. Translated from the German in 2008 by Larissa Borissova and Dmitri Rabounski.

§1. In his study on the motion of the perihelion of Mercury (see his presentation given on November 18, 1915*) Einstein set up the following problem: a point moves according to the requirement

$$\left. \begin{aligned} \delta \int ds &= 0, \\ \text{where } ds &= \sqrt{\Sigma g_{\mu\nu} dx^\mu dx^\nu}, \quad \mu, \nu = 1, 2, 3, 4 \end{aligned} \right\}, \quad (1)$$

where $g_{\mu\nu}$ are functions of the variables x , and, in the framework of this variation, these variables are fixed in the start and the end of the path of integration. Hence, in short, this point moves along a geodesic line, where the manifold is characterized by the line-element ds .

Taking this variation gives the equations of this point

$$\frac{d^2 x^\alpha}{ds^2} = \sum_{\mu, \nu} \Gamma_{\mu\nu}^\alpha \frac{dx^\mu}{ds} \frac{dx^\nu}{ds}, \quad \alpha, \beta = 1, 2, 3, 4, \quad (2)$$

where

$$\Gamma_{\mu\nu}^\alpha = -\frac{1}{2} \sum_{\beta} g^{\alpha\beta} \left(\frac{\partial g_{\mu\beta}}{\partial x^\nu} + \frac{\partial g_{\nu\beta}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\beta} \right), \quad (3)$$

while $g^{\alpha\beta}$, which are introduced and normed with respect to $g_{\alpha\beta}$, mean the reciprocal determinant[†] to the determinant $|g_{\mu\nu}|$.

*Schwarzschild means the article: Einstein A. Erklärung der Perihelbewegung der Merkur aus der allgemeinen Relativitätstheorie. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften*, 1915, S. 831–839. — Editor's comment. D.R.

[†]This is the determinant of the reciprocal matrix, i.e. a matrix whose indices are raised to the given matrix. One referred to the reciprocal matrix as the *sub-determinant*, in those years. — Editor's comment. D.R.

Commencing now and so forth, according to Einstein's theory, a test-particle moves in the gravitational field of the mass located at the point $x^1 = x^2 = x^3 = 0$, if the "components of the gravitational field" Γ satisfy the "field equations"

$$\sum_{\alpha} \frac{\partial \Gamma_{\mu\nu}^{\alpha}}{\partial x^{\alpha}} + \sum_{\alpha\beta} \Gamma_{\mu\beta}^{\alpha} \Gamma_{\nu\alpha}^{\beta} = 0 \quad (4)$$

everywhere except the point $x^1 = x^2 = x^3 = 0$ itself, and also if the determinant equation

$$|g_{\mu\nu}| = -1 \quad (5)$$

is satisfied.

These field equations in common with the determinant equation possess the fundamental property, according to which their form remains unchanged in the framework of substitution of any other variables instead of x^1, x^2, x^3, x^4 , if the substitution of the determinant equals 1.

Assume the curvilinear coordinates x^1, x^2, x^3 , while x^4 is time. We assume that the mass located at the origin of the coordinates remains unchanged with time, and also the motion is uniform and linear up to infinity. In such a case, according to the calculation by Einstein (see page 833*) the following requirements should be satisfied:

1. All the components should be independent of the time coordinate x^4 ;
2. The equalities $g_{\rho 4} = g_{4\rho} = 0$ are satisfied exactly for $\rho = 1, 2, 3$;
3. The solution is spatially symmetric at the origin of the coordinate frame in that sense that it comes to the same solution after the orthogonal transformation (rotation) of x^1, x^2, x^3 ;
4. These $g_{\mu\nu}$ vanish at infinity, except the next four boundary conditions, which are nonzero

$$g_{44} = 1, \quad g_{11} = g_{22} = g_{33} = -1.$$

The task is to find such a line-element, possessing such coefficients, that the field equations, the determinant equation, and these four requirements would be satisfied.

§2. Einstein showed that this problem in the framework of the first order approximation leads to Newton's law, and also that the second order approximation covers the anomaly in the motion of the perihelion

*Schwarzschild means page 833 in the aforementioned Einstein paper of 1915 published in *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften*. — Editor's comment. D.R.

of Mercury. The following calculation provides an exact solution of this problem. As supposed, in any case, an exact solution should have a simple form. It is important that the resulting calculation shows the uniqueness of this solution, while Einstein's approach gives ambiguity, and also that the method shown below gives (with some difficulty) the same good approximation. The following text leads to the representation of Einstein's result with increasing precision.

§3. We denote time t , while the rectangular coordinates* are denoted x, y, z . Thus the well-known line-element, satisfying the requirements 1–3, has the obvious form

$$ds^2 = F dt^2 - G (dx^2 + dy^2 + dz^2) - H (xdx + ydy + zdz)^2,$$

where F, G, H are functions of $r = \sqrt{x^2 + y^2 + z^2}$.

The condition (4) requires, at $r = \infty$: $F = G = 1, H = 0$.

Moving to the spherical coordinates† $x = r \sin \vartheta \cos \varphi, y = r \sin \vartheta \sin \varphi, z = r \cos \vartheta$, the same line-element is

$$\begin{aligned} ds^2 &= F dt^2 - G (dr^2 + r^2 d\vartheta^2 + r^2 \sin^2 \vartheta d\varphi^2) - Hr^2 dr^2 = \\ &= F dt^2 - (G + Hr^2) dr^2 - Gr^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2). \end{aligned} \quad (6)$$

In the spherical coordinates the volume element is $r^2 \sin \vartheta dr d\vartheta d\varphi$, the determinant of transformation from the old coordinates to the new ones $r^2 \sin \vartheta$ differs from 1; the field equations are still to be unchanged and, with use the spherical coordinates, we need to process complicated transformations. However the following simple method allows us to avoid this difficulty. Assume

$$x^1 = \frac{r^3}{3}, \quad x^2 = -\cos \vartheta, \quad x^3 = \varphi, \quad (7)$$

then the equality $r^2 dr \sin \vartheta d\vartheta d\varphi = dx^1 dx^2 dx^3$ is true in the whole volume element. These new variables also represent spherical coordinates in the framework of this unit determinant. They have obvious advantages to the old spherical coordinates in this problem, and, at the same time, they still remain valid in the framework of the considerations. In addition to these, assuming $t = x^4$, the field equations and the determinant equation remain unchanged in form.

*The Cartesian coordinates. — Editor's comment. D.R.

†In the original — “polar coordinates”. However it is obvious that Schwarzschild means the three-dimensional spherical coordinates. — Editor's comment. D.R.

In these new spherical coordinates the line-element has the form

$$ds^2 = F(dx^4)^2 - \left(\frac{G}{r^4} + \frac{H}{r^2} \right) (dx^1)^2 - Gr^2 \left[\frac{(dx^2)^2}{1 - (x^2)^2} + (dx^3)^2 [1 - (x^2)^2] \right], \quad (8)$$

on the basis of which we write

$$ds^2 = f_4(dx^4)^2 - f_1(dx^1)^2 - f_2 \frac{(dx^2)^2}{1 - (x^2)^2} - f_3(dx^3)^2 [1 - (x^2)^2]. \quad (9)$$

In such a case $f_1, f_2 = f_3, f_4$ are three functions of x^1 , which satisfy the following conditions

1. At $x^1 = \infty$: $f_1 = \frac{1}{r^4} = (3x^1)^{-\frac{4}{3}}$, $f_2 = f_3 = r^2 = (3x^1)^{\frac{2}{3}}$, $f_4 = 1$;
2. The determinant equation $f_1 \cdot f_2 \cdot f_3 \cdot f_4 = 1$;
3. The field equations;
4. The function f is continuous everywhere except $x^1 = 0$.

§4. To obtain the field equations we need first to construct the components of the gravitational field according to the line-element (9). The simplest way to do this is by directly taking the variation, which gives the differential equations of the geodesic line, then the components will be seen from the equations. The differential equations of the geodesic line along the line-element (9) are obtained by directly taking this variation in the form

$$\begin{aligned} f_1 \frac{d^2 x^1}{ds^2} + \frac{1}{2} \frac{\partial f_4}{\partial x^1} \left(\frac{dx^4}{ds} \right)^2 + \frac{1}{2} \frac{\partial f_1}{\partial x^1} \left(\frac{dx^1}{ds} \right)^2 - \\ - \frac{1}{2} \frac{\partial f_2}{\partial x^1} \left[\frac{1}{1 - (x^2)^2} \left(\frac{dx^2}{ds} \right)^2 + [1 - (x^2)^2] \left(\frac{dx^3}{ds} \right)^2 \right] = 0, \\ \frac{f_2}{1 - (x^2)^2} \frac{d^2 x^2}{ds^2} + \frac{\partial f_2}{\partial x^1} \frac{1}{1 - (x^2)^2} \frac{dx^1}{ds} \frac{dx^2}{ds} + \\ + \frac{f_2 x^2}{[1 - (x^2)^2]^2} \left(\frac{dx^2}{ds} \right)^2 + f_2 x^2 \left(\frac{dx^3}{ds} \right)^2 = 0, \\ f_2 [1 - (x^2)^2] \frac{d^2 x^3}{ds^2} + \frac{\partial f_2}{\partial x^1} [1 - (x^2)^2] \frac{dx^1}{ds} \frac{dx^3}{ds} - 2f_2 x^2 \frac{dx^2}{ds} \frac{dx^3}{ds} = 0, \\ f_4 \frac{d^2 x^4}{ds^2} + \frac{\partial f_4}{\partial x^1} \frac{dx^1}{ds} \frac{dx^4}{ds} = 0. \end{aligned}$$

Comparing these equations to (2) gives the components of the gravitational field

$$\begin{aligned}\Gamma_{11}^1 &= -\frac{1}{2} \frac{1}{f_1} \frac{\partial f_1}{\partial x^1}, & \Gamma_{22}^1 &= +\frac{1}{2} \frac{1}{f_1} \frac{\partial f_2}{\partial x^1} \frac{1}{1-(x^2)^2}, \\ \Gamma_{33}^1 &= +\frac{1}{2} \frac{1}{f_1} \frac{\partial f_2}{\partial x^1} [1-(x^2)^2], \\ \Gamma_{44}^1 &= -\frac{1}{2} \frac{1}{f_1} \frac{\partial f_4}{\partial x^1}, \\ \Gamma_{21}^2 &= -\frac{1}{2} \frac{1}{f_2} \frac{\partial f_2}{\partial x^1}, & \Gamma_{22}^2 &= -\frac{x^2}{1-(x^2)^2}, & \Gamma_{33}^2 &= -x^2 [1-(x^2)^2], \\ \Gamma_{31}^3 &= -\frac{1}{2} \frac{1}{f_2} \frac{\partial f_2}{\partial x^1}, & \Gamma_{32}^3 &= +\frac{x^2}{1-(x^2)^2}, \\ \Gamma_{41}^4 &= -\frac{1}{2} \frac{1}{f_4} \frac{\partial f_4}{\partial x^1},\end{aligned}$$

while the rest of the components of it are zero.

Due to the symmetry of rotation around the origin of the coordinates, it is sufficient to construct the field equations at only the equator ($x^2=0$): once they are differentiated, we can substitute 1 instead of $1-(x^2)^2$ everywhere into the above obtained formulae. Thus, after this algebra, we obtain the field equations

$$\begin{aligned}a) \quad \frac{\partial}{\partial x^1} \left(\frac{1}{f_1} \frac{\partial f_1}{\partial x^1} \right) &= \frac{1}{2} \left(\frac{1}{f_1} \frac{\partial f_1}{\partial x^1} \right)^2 + \left(\frac{1}{f_2} \frac{\partial f_2}{\partial x^1} \right)^2 + \frac{1}{2} \left(\frac{1}{f_4} \frac{\partial f_4}{\partial x^1} \right)^2, \\ b) \quad \frac{\partial}{\partial x^1} \left(\frac{1}{f_1} \frac{\partial f_2}{\partial x^1} \right) &= 2 + \frac{1}{f_1 f_2} \left(\frac{\partial f_2}{\partial x^1} \right)^2, \\ c) \quad \frac{\partial}{\partial x^1} \left(\frac{1}{f_1} \frac{\partial f_4}{\partial x^1} \right) &= \frac{1}{f_1 f_4} \left(\frac{\partial f_4}{\partial x^1} \right)^2.\end{aligned}$$

Besides these three equations, the functions f_1, f_2, f_3 should satisfy the determinant equation

$$d) \quad f_1(f_2)^2 f_4 = 1 \quad \text{or} \quad \frac{1}{f_1} \frac{\partial f_1}{\partial x^1} + \frac{2}{f_2} \frac{\partial f_2}{\partial x^1} + \frac{1}{f_4} \frac{\partial f_4}{\partial x^1} = 0.$$

First of all I remove $b)$. So three functions f_1, f_2, f_4 of $a), c)$, and $d)$ still remain. The equation $c)$ takes the form

$$c') \quad \frac{\partial}{\partial x^1} \left(\frac{1}{f_4} \frac{\partial f_4}{\partial x^1} \right) = \frac{1}{f_1 f_4} \frac{\partial f_1}{\partial x^1} \frac{\partial f_4}{\partial x^1}.$$

Integration of it gives

$$c'') \quad \frac{1}{f_4} \frac{\partial f_4}{\partial x^1} = \alpha f_1,$$

where α is the constant of integration. Summation of $a)$ and $c')$ gives

$$\frac{\partial}{\partial x^1} \left(\frac{1}{f_1} \frac{\partial f_1}{\partial x^1} + \frac{1}{f_4} \frac{\partial f_4}{\partial x^1} \right) = \left(\frac{1}{f_2} \frac{\partial f_2}{\partial x^1} \right)^2 + \frac{1}{2} \left(\frac{1}{f_1} \frac{\partial f_1}{\partial x^1} + \frac{1}{f_4} \frac{\partial f_4}{\partial x^1} \right)^2.$$

With taking $d)$ into account, it follows that

$$-2 \frac{\partial}{\partial x^1} \left(\frac{1}{f_2} \frac{\partial f_2}{\partial x^1} \right) = 3 \left(\frac{1}{f_2} \frac{\partial f_2}{\partial x^1} \right)^2.$$

After integration, we obtain

$$\frac{1}{\frac{1}{f_2} \frac{\partial f_2}{\partial x^1}} = \frac{3}{2} x^1 + \frac{\rho}{2},$$

where ρ is the constant of integration. Or

$$\frac{1}{f_2} \frac{\partial f_2}{\partial x^1} = \frac{2}{3x^1 + \rho}.$$

We integrate it once again:

$$f_2 = \lambda (3x^1 + \rho)^{\frac{2}{3}},$$

where λ is the constant of integration. The condition at infinity requires: $\lambda = 1$. Hence

$$f_2 = (3x_1 + \rho)^{\frac{2}{3}}. \quad (10)$$

Next, it follows from $c'')$ and $d)$ that

$$\frac{\partial f_4}{\partial x^1} = \alpha f_1 f_4 = \frac{\alpha}{(f_2)^2} = \frac{\alpha}{(3x_1 + \rho)^{\frac{4}{3}}}.$$

We integrate it, taking the condition at infinity into account:

$$f_4 = 1 - \alpha (3x^1 + \rho)^{-\frac{1}{3}}. \quad (11)$$

Finally, it follows from $d)$ that

$$f_1 = \frac{(3x^1 + \rho)^{-\frac{4}{3}}}{1 - \alpha (3x^1 + \rho)^{-\frac{1}{3}}}. \quad (12)$$

As easy to check, the equation $b)$ corresponds to the found formulae for f_1 and f_2 .

This satisfies all the requirements up to the continuity condition. The function f_1 remains continuous, if

$$1 = \alpha (3x^1 + \rho)^{-\frac{1}{3}}, \quad 3x^1 = \alpha^3 - \rho.$$

In order to break the continuity at the origin of the coordinates, there should be

$$\rho = \alpha^3. \quad (13)$$

The continuity condition connects, by the the same method, both constants of integration ρ and α .

Now, the complete solution of our problem has the form

$$f_1 = \frac{1}{R^4} \frac{1}{1 - \frac{\alpha}{R}}, \quad f_2 = f_3 = R^2, \quad f_4 = 1 - \frac{\alpha}{R},$$

where the auxiliary quantity R has been introduced

$$R = (3x^1 + \rho)^{\frac{1}{3}} = (r^3 + \alpha^3)^{\frac{1}{3}}.$$

If substituting the formulae of these functions f into the formula of the line-element (9), and coming back to the regular spherical coordinates, we arrive at such a formula for the line-element

$$\left. \begin{aligned} ds^2 = \left(1 - \frac{\alpha}{R}\right) dt^2 - \frac{dR^2}{1 - \frac{\alpha}{R}} - R^2 (d\vartheta^2 + \sin^2\vartheta d\varphi^2) \\ R = (r^3 + \alpha^3)^{\frac{1}{3}} \end{aligned} \right\}, \quad (14)$$

which is the exact solution of the Einstein problem.

This formula contains the sole constant of integration α , which is dependent on the numerical value of the mass located at the origin of the coordinates.

§5. The uniqueness of this solution follows from the aforementioned calculations. For one who is troubled with the uniqueness of Einstein's method, followed from this, we consider the following example. There above, from the continuity condition, the formula

$$f_1 = \frac{(3x^1 + \rho)^{-\frac{4}{3}}}{1 - \alpha (3x^1 + \rho)^{-\frac{1}{3}}} = \frac{(r^3 + \rho)^{-\frac{4}{3}}}{1 - \alpha (r^3 + \rho)^{-\frac{1}{3}}}$$

was obtained. In the case where α and ρ are small values, the second order term and the higher order terms vanish from the series so that

$$f_1 = \frac{1}{r^4} \left[1 + \frac{\alpha}{r} - \frac{4}{3} \frac{\rho}{r^3} \right].$$

This exception, in common with the respective exceptions for f_1 , f_2 , f_4 taken to within the same precision, satisfies all the requirements of this problem. The continuity requirement added nothing in the framework of this approximation, but only a break at the point of the origin of the coordinates. Both constants α and ρ are arbitrarily determined, so the physical side of this problem is not determined. The exact solution of this problem manifests that in a real case, with the approximations, a break appears not at the point of the origin of the coordinates, but in the region $r = (\alpha^3 - \rho)^{\frac{1}{3}}$, and we should suppose $\rho = \alpha^3$ in order to move the break to the origin of the coordinates. In the framework of such an approximation through the exponents of α and ρ , we need to know very well the law which rules these coefficients, and also be masters in the whole situation, in order to understand the necessity of connexion between α and ρ .

§6. In the end, we are looking for the equation of a point moving along the geodesic line in the gravitational field related to the line-element (14). Proceeding from the three circumstances according to which the line-element is homogeneous, differentiable, and its coefficients are independent of t and ρ , we take the variation so we obtain three intermediate integrals. Because the motion is limited to the equatorial plane ($\vartheta = 90^\circ$, $d\vartheta = 0$), these intermediate integrals have the form

$$\left(1 - \frac{\alpha}{R}\right) \left(\frac{dt}{ds}\right)^2 - \frac{1}{1 - \frac{\alpha}{R}} \left(\frac{dR}{ds}\right)^2 - R^2 \left(\frac{d\varphi}{ds}\right)^2 = \text{const} = h, \quad (15)$$

$$R^2 \frac{d\varphi}{ds} = \text{const} = c, \quad (16)$$

$$\left(1 - \frac{\alpha}{R}\right) \frac{dt}{ds} = \text{const} = 1, \quad (17)$$

where the third integral means definition of the unit of time.

From here it follows that

$$\left(\frac{dR}{d\varphi}\right)^2 + R^2 \left(1 - \frac{\alpha}{R}\right) = \frac{R^4}{c^2} \left[1 - h \left(1 - \frac{\alpha}{R}\right)\right]$$

or, for $\frac{1}{R} = x$,

$$\left(\frac{dx}{d\varphi}\right)^2 = \frac{1-h}{c^2} + \frac{h\alpha}{c^2}x - x^2 + \alpha x^3. \quad (18)$$

We denote $\frac{c^2}{h} = B$, $\frac{1-h}{h} = 2A$ that is identical to Einstein's equations (11) in the cited presentation*, and gives the observed anomaly of the perihelion of Mercury.

In a general case Einstein's approximation for a curved trajectory meets the exact solution, only if we introduce

$$R = (r^3 + \alpha^3)^{\frac{1}{3}} = r \left(1 + \frac{\alpha^3}{r^3}\right)^{\frac{1}{3}} \quad (19)$$

instead of r . Because $\frac{\alpha}{r}$ is close to twice the square of the velocity of the planet (the velocity of light is 1), the expression within the brackets, in the case of Mercury, is different from 1 by a value of the order 10^{-12} . The quantities R and r are actually identical, so Einstein's approximation satisfies the practical requirements of even very distant future.

In the end it is required to obtain the exact form of Kepler's third law for circular trajectories. Given an angular velocity $n = \frac{d\varphi}{dt}$, according to (16) and (17), and introducing $x = \frac{1}{R}$, we have

$$n = cx^2(1 - \alpha x).$$

In a circle both $\frac{dx}{d\varphi}$ and $\frac{d^2x}{d\varphi^2}$ should be zero. This gives, according to (18), that

$$\frac{1-h}{c^2} + \frac{h\alpha}{c^2}x - x^2 + \alpha x^3 = 0, \quad \frac{h\alpha}{c^2} - 2x + 3\alpha x^2 = 0.$$

Removing h from both circles gives

$$\alpha = 2c^2x(1 - \alpha x)^2.$$

From here it follows that

$$n^2 = \frac{\alpha}{2}x^3 = \frac{\alpha}{2R^3} = \frac{\alpha}{2(r^3 + \alpha^3)}.$$

Deviation of this formula from Kepler's third law is absolutely invisible up to the surface of the Sun. However given an ideal point-mass, the

*Einstein A. Erklärung der Perihelbewegung der Merkur aus der allgemeinen Relativitätstheorie. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften*, 1915, S. 831–839. — Editor's comment. D.R.

angular velocity does not experience unbounded increase with lowering of the orbital radius (such an unbounded increase should be experienced according to Newton's law), but approximates to a finite limit

$$n_0 = \frac{1}{\alpha\sqrt{2}}.$$

(For a mass which is in the order of the mass of the Sun, this boundary frequency should be about 10^4 per second.) This circumstance should be interesting in the case where a similar law rules the molecular forces.

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